DIFFRACTION OF A PLANE HYDRO-ACOUSTIC WAVE AT A CRACK IN AN ELASTIC PLATE

(DIFRAKTSIIA PLOSKOI GIDROAKUSTICHESKOI VOLNY NA TRESHCHINE V UPRUGOI PLASTINE)

PMM Vol.27, No.6, 1963, pp.1037-1043

D. P. KOUZOV
(Leningrad)

(Received March 14, 1963)

The two-dimensional steady problem of the diffraction of a plane hydro-acoustic wave at the junction of two semi-infinite planes covering the surface of a fluid was treated in [1]. In that paper the "general" solution of the problem was constructed (i.e. the solution irrespective of the conditions at the junction of the plate), and that solution contained four arbitrary constants.

The present paper considers a particular case of the aforementioned problem of great interest. An infinitesimally thin crack is assumed to separate the plates, and the elastic characteristics of both plates are taken to be identical. In the first section the solution of the problem will be described. The second section is devoted to the investigation of the solution as $kh \to 0$ (where k is the wave number in the fluid and h is the thickness of the plates).

1. Formulation and solution of the problem. An infinite horizontal elastic plate is divided into two identical parts by an infinitesimally thin rectilinear crack. The half-space below the plate is occupied by an ideal compressible fluid. A plane monochromatic acoustic wave originating in the depths of the fluid propagates normal to the crack. We seek the diffraction field caused by this wave.

With an appropriate choice of coordinate axes (Fig. 1), this problem is two-dimensional. It reduces to the determination of the function $U(x, y)$ (the acoustic potential), which is continuous up to the x-axis, and which satisfies the Helmholtz equation

$$\triangle U + k^2U = 0 \qquad (-\infty < x < +\infty,\ 0 < y < +\infty) \qquad (1.1)$$

the boundary conditions

$$\left(\frac{\partial^5 U}{\partial x^4 \partial y} - \delta \frac{\partial U}{\partial y} + \nu U\right)\Big|_{y=0} = 0 \qquad (x \neq 0) \qquad (1.2)$$

and the boundary-contact conditions

$$\lim_{x \to +0} \frac{\partial^3 U\,(x,\,0)}{\partial x^2\, \partial y} = 0, \qquad \lim_{x \to +0} \frac{\partial^4 U\,(x,\,0)}{\partial x^3\, \partial y} = 0 \qquad (1.3)$$

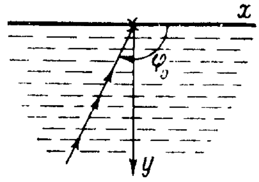

Fig. 1.

The principle of maximum absorption must be fulfilled here for the difference $U - U_0$, where U_0 is the incident wave

$$U_0 = A \exp\,(i\varkappa x - i\sqrt{k^2 - \varkappa^2}\,y), \qquad (\varkappa = -k\cos\varphi_0 > 0) \qquad (1.4)$$

The angle of incidence φ_0 is measured from the positive x-axis.

The notations

$$\delta = \frac{12\,(1-\sigma^2)}{E}\,\rho_0 c^2\,\frac{k^2}{h^2}, \qquad \nu = \frac{12\,(1-\sigma^2)}{E}\,\rho c^2\,\frac{k^2}{h^3}$$

were introduced in conditions (1.2).

Here E is Young's modulus, σ is Poisson's ratio, h is the thickness of the plate, ρ_0 is the density of the plate material, ρ is the density of the fluid and c is the velocity of sound in the fluid.

We make use of the more general solution of this problem which was deduced in [1] (for constants ν and δ different for $x > 0$ and $x < 0$, and unspecified boundary-contact conditions (1.3)). For the case under consideration we obtain

$$U = U_0 + U_1 + W \qquad (1.5)$$

$$U_1 = A\,\frac{(\varkappa^4 - \delta)\sqrt{k^2 - \varkappa^2} + i\nu}{(\varkappa^4 - \delta)\sqrt{k^2 - \varkappa^2} - i\nu}\,\exp\,(i\varkappa x + i\sqrt{k^2 - \varkappa^2}\,y) \qquad (1.6)$$

$$W = \frac{A}{2\pi i}\int_{-\infty}^{\infty} \frac{a\lambda^3 + b\lambda^2 + c\lambda + d}{(\lambda^4 - \delta)\sqrt{k^2 - \lambda^2} - i\nu}\,\exp\,(i\lambda x + i\sqrt{k^2 - \lambda^2}\,y)\,d\lambda \qquad (1.7)$$

where U_0 is the incident wave (see (1.4)), U_1 is the reflected wave and W the diffracted disturbance. The radical $\sqrt{(k^2 - \lambda^2)}$ is considered to be positive on the segment $(-k, k)$; the choice of its branch on the remaining portions of the path of integration is clear from Fig. 2, in which the cuts are depicted by dashed lines, while the contour of integration is shown by a heavy solid line. The denominator of the integrand in (1.7) has ten roots $\pm\lambda_0, \pm\lambda_1, \ldots, \pm\lambda_4$ on a two-sheeted Riemann surface (the notation for the roots has been changed from [1]). The roots $\pm\lambda_0$, $\pm\lambda_1$ and $\pm\lambda_2$ are located on the main sheet of the Riemann surface, the real roots being $\pm\lambda_0$. We note that the integral (1.7) is not improper at finite points: the contour of integration avoids the points $\pm\lambda_0$, as shown in Fig. 2.

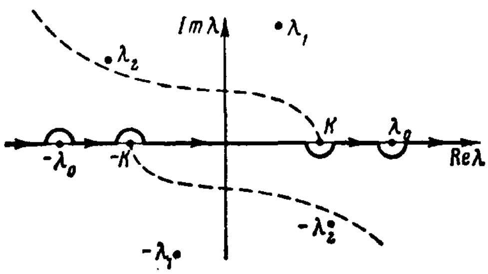

Fig. 2.

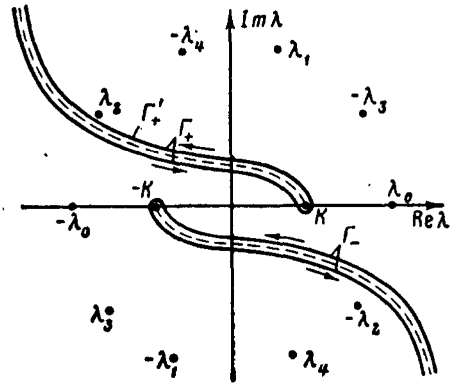

Fig. 3.

The constants a, b, c and d are determined from the four boundary-contact conditions (1.3). We represent these conditions in the following form:

$$\left.\frac{\partial^3 W}{\partial x^2\,\partial y}\right|_{\substack{y=0\\x=\pm 0}} = -A\,\frac{2\nu\varkappa^2\sqrt{k^2-\varkappa^2}}{(\varkappa^4-\delta)\sqrt{k^2-\varkappa^2}-i\nu}$$

$$\left.\frac{\partial^4 W}{\partial y^3\,\partial y}\right|_{\substack{y=0\\x=\pm 0}} = -A\,\frac{2i\nu\varkappa^3\sqrt{k^2-\varkappa^2}}{(\varkappa^4-\delta)\sqrt{k^2-\varkappa^2}-i\nu} \qquad (1.8)$$

We will now carry out the calculations for the first of relations (1.8). The problem consists of passing to the limits $y \to 0$, $x \to +0$ in that sequence in the expressions

$$\frac{\partial^3 W}{\partial x^2\,\partial y} = \frac{A}{2\pi i}\int\limits_{-\infty}^{\infty}\frac{a\lambda^3+b\lambda^2+c\lambda+d}{(\lambda^4-\delta)\sqrt{k^2-\lambda^2}-i\nu}\times$$

$$\times\left(-i\lambda^2\sqrt{k^2-\lambda^2}\right)\exp\left(i\lambda x+i\sqrt{k^2-\lambda^2}\,y\right)d\lambda \qquad (1.9)$$

The first limiting process, $y \to 0$, may be carried out immediately under the integral sign; [1] described how to treat the divergent integral arising in this process.

Considering x to be positive, we now deform the contour of integration into the loop Γ_+ surrounding the upper branch cut (Fig. 3). Calculating the residues at the poles λ_0, λ_1 and λ_2 which are crossed in this process, we have

$$\left.\frac{\partial^3 W}{\partial x^2\,\partial y}\right|_{\substack{y=0\\x>0}} = \frac{A}{2\pi i}\int_{\Gamma_+}\frac{a\lambda^3+b\lambda^2+c\lambda+d}{(\lambda^4-\delta)\sqrt{k^2-\lambda^2}-iv}\left(-i\lambda^2\sqrt{k^2-\lambda^2}\right)e^{i\lambda x}\,d\lambda -$$
$$- iA\sum_{s=0}^{2}\frac{a\lambda_s^4+b\lambda_s^3+c\lambda_s^2+d\lambda_s}{5\lambda_s^4-4\lambda_s^2k^2-\delta}\left(\lambda_s^2-k^2\right)$$

The integral along the whole loop Γ_+ is reduced to the integral along only the right-hand side Γ_+' of the cut.

Since the degree of the algebraic expression in front of the exponential in the integrand is reduced by this means, then the second limiting process, $x \to +0$, may be carried out under the integral

$$\left.\frac{\partial^3 W}{\partial x^2\,\partial y}\right|_{\substack{y=0\\x=+0}} = \frac{A\nu}{\pi}\int_{\Gamma_+'}\frac{a\lambda^5+b\lambda^4+c\lambda^3+d\lambda^2}{(\lambda^4-\delta)^2(\lambda^2-k^2)-\nu^2}\sqrt{\lambda^2-k^2}\,d\lambda -$$
$$- iA\sum_{s=0}^{2}\frac{a\lambda_s^4+b\lambda_s^3+c\lambda_s^2+d\lambda_s}{5\lambda_s^4-4\lambda_s^2k^2-\delta}\left(\lambda_s^2-k^2\right) \tag{1.10}$$

The radical $\sqrt{(\lambda^2 - k^2)}$ is taken as positive on the segment (k, ∞) of the real axis.

The integral in (1.10) may be expressed in terms of elementary functions. For this we reduce the integral along Γ_+' to an integral along the entire contour Γ, consisting of the two loops Γ_+ and Γ_- (see Fig. 3), with the help of the equalities

$$\int_{\Gamma_+'}\frac{\lambda^{2n+1}\sqrt{\lambda^2-k^2}\,d\lambda}{(\lambda^4-\delta)^2(\lambda^2-k^2)-\nu^2} = \frac{1}{4}\int_{\Gamma}\frac{\lambda^{2n+1}\sqrt{\lambda^2-k^2}\,d\lambda}{(\lambda^4-\delta)^2(\lambda^2-k^2)-\nu^2}$$

$$\int_{\Gamma_+'}\frac{\lambda^{2n}\sqrt{\lambda^2-k^2}\,d\lambda}{(\lambda^4-\delta)^2(\lambda^2-k^2)-\nu^2} = +\frac{1}{2\pi i}\int_{\Gamma}\frac{\lambda^{2n}\sqrt{\lambda^2-k^2}}{(\lambda^4-\delta)^2(\lambda^2-k^2)-\nu^2}\ln\frac{\lambda+\sqrt{\lambda^2-k^2}}{k}\,d\lambda$$

In the lower formula the logarithm is to be taken as positive for values $\lambda \in (k, \infty)$.

It is clear that the integral along Γ may be replaced by the sum of the residues of the integrand at the poles lying on the sheet of the Riemann surface under consideration. As a result we obtain

$$\left.\frac{\partial^3 W}{\partial x^2\,\partial y}\right|_{\substack{y=0\\x=\pm 0}} = \frac{A}{\pi}\sum_{s=0}^{4}\frac{(b\lambda_s^3+d\lambda_s)(\lambda_s^2-k^2)}{5\lambda_s^4-4\lambda_s^2k^2-\delta}\left\{\ln\left[\frac{\lambda_s}{k}\left(1+\sqrt{1-\frac{k^2}{\lambda_s^2}}\right)\right]-i\frac{\pi}{2}\right\}\mp$$

$$\mp\frac{iA}{2}\sum_{s=0}^{4}\frac{(a\lambda_s^4+c\lambda_s^2)(\lambda_s^2-k^2)}{5\lambda_s^4-4\lambda_s^2k^2-\delta} \tag{1.11}$$

Here and everywhere in the following equations the choice of the logarithm is fixed by the requirements

$$0<\operatorname{Re}\ln\left[\frac{\lambda_s}{k}\left(1+\sqrt{1-\frac{k^2}{\lambda_s^2}}\right)\right]$$

$$0\leqslant\operatorname{Im}\ln\left[\frac{\lambda_s}{k}\left(1+\sqrt{1-\frac{k^2}{\lambda_s^2}}\right)\right]<2\pi$$

Two signs are given in expression (1.11). The upper sign corresponds to calculations carried out for $x\to+0$. The lower sign corresponds to the case $x\to-0$, and this result is obtained from (1.9) by replacing λ by $-\lambda$.

Now it is clear that the first two equalities of system (1.8) may be simplified by termwise subtraction and addition. The second pair of equalities of this system are transformed in a similar manner. Thus the system of equations for a, b, c and d takes the form

$$\sum_{s=0}^{4}\frac{(a\lambda_s^4+c\lambda_s^2)(\lambda_s^2-k^2)}{5\lambda_s^4-4\lambda_s^2k^2-\delta}=0$$

$$\sum_{s=0}^{4}\frac{(a\lambda_s^5+c\lambda_s^3)(\lambda_s^2-k^2)}{5\lambda_s^4-4\lambda_s^2k^2-\delta}\left\{\ln\left[\frac{\lambda_s}{k}\left(1+\sqrt{1-\frac{k^2}{\lambda_s^2}}\right)\right]-i\frac{\pi}{2}\right\}=$$

$$=\frac{2\pi\nu\varkappa^3\sqrt{k^2-\varkappa^2}}{(\varkappa^4-\delta)\sqrt{k^2-\varkappa^2}-i\nu}$$

$$\sum_{s=0}^{4}\frac{(b\lambda_s^4+d\lambda_s^2)(\lambda_s^2-k^2)}{5\lambda_s^4-4\lambda_s^2k^2-\delta}=0 \tag{1.12}$$

$$\sum_{s=0}^{4}\frac{(b\lambda_s^3+d\lambda_s)(\lambda_s^2-k^2)}{5\lambda_s^4-4\lambda_s^2k^2-\delta)}\left\{\ln\left[\frac{\lambda_s}{k}\left(1+\sqrt{1-\frac{k^2}{\lambda^2_s}}\right)\right]-i\frac{\pi}{2}\right\}=$$

$$=\frac{2\pi\nu\varkappa^2\sqrt{k^2-\varkappa^2}}{(\varkappa^4-\delta)\sqrt{k^2-\varkappa^2}-i\nu}$$

The formulas (1.4), (1.5), (1.6), (1.7) and (1.12) give the solution of the formulated problem.

System (1.12) may be easily solved numerically for given values of the physical parameters E, σ, ρ, ρ_0, h and k. The roots of the equation

$$(\lambda_s^4 - \delta)^2 (\lambda_s^2 - k^2) - \nu^2 = 0 \tag{1.13}$$

required for these calculations may be tabulated. Such calculations were carried out under the direction of V.Iu. Zavadskii in the Institute of Acoustics of the Academy of Sciences, USSR.

Some equalities which are not tied to a particular choice of parameters will be deduced below.

2. Asymptotic investigation of the solution. The problem under consideration pertains to small wave numbers. In order to simulate an elastic layer by an infinitesimally thin plate, we must assume that $kh \ll 1$. Thus kh is a small dimensionless parameter of the problem. We study the behavior of the diffraction field as $kh \to 0$.

We introduce the dimensionless variables $\xi = x/h$, $\eta = y/h$, $\mu = \lambda h$ and transform equalities (1.7), (1.12) and (1.13), which specify the diffraction field, into the following form:

$$W = \frac{A}{2\pi i} \int_{-\infty}^{\infty} \frac{a_0\mu^3 + b_0\mu^2 + c_0\mu + d_0}{(\mu^4 - \delta_0 k^2h^2)\sqrt{k^2h^2 - \mu^2} - i\nu_0 k^2h^2} \exp\left(i\mu\xi + i\sqrt{k^2h^2 - \mu^2}\,\eta\right) d\mu \tag{2.1}$$

$$\sum_{s=0}^{4} \frac{(a_0\mu_s^4 + c_0\mu_s^2)(\mu_s^2 - k^2h^2)}{5\mu_s^4 - 4\mu_s^2k^2h^2 - \delta_0 k^2h^2} = 0$$

$$\sum_{s=0}^{4} \frac{(a_0\mu_s^5 + c_0\mu_s^3)(\mu_s^2 - k^2h^2)}{5\mu_s^4 - 4\mu_s^2k^2h^2 - \delta_0 k^2h^2} \left\{\ln\left[\frac{\mu_s}{kh}\left(1 + \sqrt{1 - \frac{k^2h^2}{\mu_s^2}}\right)\right] - i\frac{\pi}{2}\right\} =$$

$$= \frac{2\pi\nu_0 \cos^3\varphi_0 \sin\varphi_0 (kh)^4}{(k^3h^3\cos^4\varphi_0 - \delta_0 kh)\sin\varphi_0 - i\nu_0}$$

$$\sum_{s=0}^{4} \frac{(b_0\mu_s^4 + d_0\mu_s^2)(\mu_s^2 - k^2h^2)}{5\mu_s^4 - 4\mu_s^2k^2h^2 - \delta_0 k^2h^2} = 0 \tag{2.2}$$

$$\sum_{s=0}^{4} \frac{(b_0\mu_s^3 + d_0\mu_s)(\mu_s^2 - k^2h^2)}{5\mu_s^4 - 4\mu_s^2k^2h^2 - \delta_0 k^2h^2} \left\{\ln\left[\frac{\mu_s}{kh}\left(1 + \sqrt{1 - \frac{k^2h^2}{\mu_s^2}}\right)\right] - i\frac{\pi}{2}\right\} =$$

$$= \frac{2\pi\nu_0 \cos^2\varphi_0 \sin\varphi_0 (kh)^3}{(k^3h^3\cos^4\varphi_0 - \delta_0 kh)\sin\varphi_0 - i\nu_0}$$

$$(\mu_s^4 - \delta_0 k^2h^2)^2 (\mu_s^2 - k^2h^2) = \nu_0^2 k^4h^4 \tag{2.3}$$

The subscript 0 refers here to the new (dimensionless) physical parameters corresponding to the old unsubscripted parameters. The parameters δ_0 and ν_0 are obtained from δ and ν by eliminating their dependence on

k and h

$$\delta_0 = \frac{12(1-\sigma^2)}{E}\rho_0 c^2 = 6(1-\sigma)\frac{c^2}{c_t^2}, \quad \nu_0 = \frac{12(1-\sigma^2)}{E}\rho c^2 = 6(1-\sigma)\frac{\rho c^2}{\rho_0 c_t^2}$$

$$\left(c_t = \sqrt{\frac{E}{2\rho_0(1+\sigma)}}\right)$$

Here c_t is the velocity of transverse waves in the plate material.

Using (2.3) and (2.4), we may expand the unknowns a_0, b_0, c_0 and d_0 in Taylor series in certain fractional powers of the parameter kh. Below we will restrict ourselves to only the first terms of such expansions. With the change of variables

$$\mu_s = M_s\,(kh)^{3/5}$$

equation (2.3) reduces to the equation

$$[M_s^4 - \delta_0\,(kh)^{2/5}]^2\,[M_s^4 - (kh)^{6/5}] = \nu_0^2$$

the roots M_s of which may be easily found by a Taylor expansion in powers of $(kh)^{2/5}$.

In subsequent calculations appear the expansions for μ_s, as well as for certain simple functions of μ_s. We cite for example only the representation most frequently encountered

$$\mu_s^2 = \nu_0^{2/5}\Big[e^{4/5\,\pi is}\,(kh)^{4/5} + \frac{2}{5}e^{-4/5\,\pi is}\,\frac{\delta_0}{\nu_0^{4/5}}\,(kh)^{6/5} -$$
$$- \frac{1}{25}e^{-11/5\,\pi is}\,\frac{\delta_0^2}{\nu_0^{8/5}}\,(kh)^{8/5} + \frac{1}{5\nu_0^{2/5}}\,(kh)^{10/5} + \ldots\Big]$$

The expansion is carried out to the fourth term, since in certain operations cancellation of the preceding terms occurs. We note in passing that as $kh \to 0$, the roots of equation (2.3) are asymptotically situated at the vertices of a regular pentagon with center at the origin

$$\mu_s \approx \nu_0^{1/5} e^{-2/5\,\pi is}\,(kh)^{2/5}$$

The calculations lead to the following expressions for the constants:

$$a_0 = 5\,(e^{6/5\,\pi i} - 1)\frac{\cos^3\varphi_0 \sin\varphi_0}{\nu_0^{3/5}}\,(kh)^{14/5}\{1 + O\,[(kh)^{2/5}\}]$$

$$b_0 = -\,5\,(e^{2/5\,\pi i} - 1)\frac{\cos^2\varphi_0 \sin\varphi_0}{\nu_0^{1/5}}\,(kh)^{13/5}\{1 + O\,[(kh)^{2/5}]\}$$

$$c_0 = \frac{2}{25}(e^{6/5\,\pi i} - 1)\,\frac{\delta_0^3 \cos^3\varphi_0 \sin\varphi_0}{\nu_0^{13/5}}\,(kh)^{24/5}\{1 + O\,[(kh)^{2/5}]\}$$

$$d_0 = -\frac{2}{25}\left(e^{2/5\,\pi i} - 1\right)\frac{\delta_0^3 \cos^2\varphi_0 \sin\varphi_0}{\nu_0^{11/5}}(kh)^{23/5}\{1 + O\,[(kh)^{2/5}]\}$$

The basic components of the diffraction field W are cylindrical and surface waves. The cylindrical wave W_0 is determined by the method of stationary phase

$$W_0 = V\,(\varphi)\frac{e^{ikr}}{\sqrt{kr}} \qquad (x = r\cos\varphi,\ y = r\sin\varphi) \tag{2.4}$$

$$V\,(\varphi) = \frac{5A}{\sqrt{2\pi}}\,e^{3/4\,\pi i}\left(e^{2/5\,\pi i} - 1\right)\frac{\cos^2\varphi_0\sin\varphi_0}{\nu_0^{4/5}}\left(\cos^2\varphi + \frac{2\delta_0^3}{125\,\nu_0^2}\right)\sin\varphi\ (kh)^{18/5}$$

In the above spirit, only the leading term in the expansion with respect to the parameter $(kh)^{2/5}$ will be retained in the present calculations. It is clear from formula (2.5) that a diagram of the directionality of the cylindrical wave consists of two petals. Maximum radiation is observed for the angles φ_1 and φ_2

$$\varphi_1 = \sin^{-1}\left(\frac{1}{3} + \frac{2\delta_0^3}{275\,\nu_0^2}\right)^{1/2}, \qquad \varphi_2 = \pi - \varphi_1$$

We note that the positions of these maxima are independent of the direction φ_0 of the incident wave. The intensity of radiation in the vertical direction is very small, and the cylindrical waves do not propagate at all in either of the horizontal directions. For the energy transmitted by the wave in unit time

$$\Pi_0 = \frac{\rho kc}{2}\int_0^\pi |V\,(\varphi)|^2\,d\varphi$$

the calculations give

$$\Pi_0 = \frac{25}{32}\left(1 - \cos\frac{2}{5}\pi\right)A^2\frac{\rho c\cos^4\varphi_0\sin^2\varphi_0}{h\nu_0^{12/5}}\left[1 + \frac{4\delta_0^3}{125\,\nu_0^2} + 2\left(\frac{2\delta_0^3}{125\,\nu_0^2}\right)^2\right](kh)^{41/5} \tag{2.5}$$

or

$$\Pi_0 \approx \frac{25}{32}\left(1 - \cos\frac{2}{5}\pi\right)A^2\frac{\rho c\cos^4\varphi_0\sin^2\varphi_0}{h\nu_0^{12/5}}(kh)^{41/5}$$

The direct and reverse surface waves W_+ and W_- are calculated by taking the residues of the integrand at $\lambda = \pm\lambda_0$ $(\mu = \pm\mu_0)$

$$W_\pm = A\,\frac{i\left(e^{2/5\pi i} - 1\right)}{\nu_0^{3/5}}\cos^2\varphi_0\,\sin\varphi_0\,\exp\left[\frac{\pm ix - y}{h}\,\nu_0^{1/5}(kh)^{2/5}\right] \tag{2.6}$$

The energy carried by each of them is

$$\Pi_{\pm} = A^2 \frac{(1 - \cos {}^2/_5\,\pi)\ \rho c \cos^4 \varphi_0 \sin^2 \varphi_0}{2h\nu_0^{4/5}} (kh)^{23/5} \tag{2.7}$$

which is considerably greater than the energy transported by the cylindrical wave.

The process of propagation of surface waves in the fluid is accompanied by vibrations in the elastic layer covering the fluid. The energy Π_+' transported in the direction of increasing x per unit time through the layer by this process may be calculated by means of the expression

$$\Pi_+' = \frac{h^2 E}{12\,kc\,(1-\sigma^2)} \operatorname{Im} \left(\frac{\partial W_+^*}{\partial y} \frac{\partial^4 W_+}{\partial x^3 \partial y} - \frac{\partial^2 W_+^*}{\partial x\,\partial y} \frac{\partial^3 W_+}{\partial x^2 \partial y} \right) \Big|_{y=0}$$

where W_+^* denotes the complex conjugate of W_+. It turns out to be equal to four times the amount of energy transported through the fluid in the same direction, i.e.

$$\Pi_{\pm}' = 4\Pi_{\pm} \tag{2.8}$$

We denote by j the amount of energy of the incident wave U_0 which falls on a unit length in the horizontal direction

$$j = A^2 \frac{k^2 \rho c}{2} \sin \varphi_0 \tag{2.9}$$

We introduce the quantity Λ, equal to the ratio of the total rate of energy flow in diffraction from the crack to the quantity j. The quantity Λ, which has the units of length, will be called the effective diameter of the crack

$$\Lambda = 10 \left(1 - \cos \frac{2}{5}\,\pi\right) \frac{\cos^4 \varphi_0 \sin \varphi_0}{\nu_0^{4/5}} (kh)^{11/5} h \tag{2.10}$$

It is clear from the formula that the diameter of the crack is small in comparison with the thickness of the plate. Thus the effect of scattering at the crack will be a weak effect and will not be accompanied by any significant expenditure of energy.

BIBLIOGRAPHY

1. Kouzov, D.P., Difraktsiia ploskoi gidroakusticheskoi volny na styke dvukh plastin (Diffraction of a plane hydro-acoustic wave at the junction of two plates). *PMM* Vol. 27, No. 3, 1963.

Translated by F.A.L.